VII. AC Machinery Fundamentals

Introduction

AC machines are AC motors and AC generators.

There are two types of AC machines:

Synchronous machines – the magnetic field current is supplied by a separate DC power source;

Induction machines – the magnetic field current is supplied by magnetic induction (transformer action) into their field windings.

The field circuits of most AC machines are located on their rotors.

Every AC (or DC) motor or generator has two parts: rotating part (**rotor**) and a stationary part (**stator**).

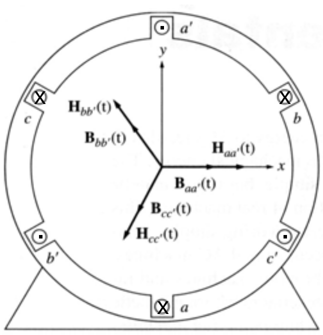
Rotating Magnetic Field

The basic idea of an electric motor is to generate two magnetic fields: rotor magnetic field and stator magnetic field and make the stator field rotating. In this situation, the rotor will constantly turning to align its magnetic field with the stator field.

The fundamental principle of AC machine operation is to make a 3-phase set of currents, each of equal magnitude and with a phase difference of 120°, to flow in a 3-phase winding. In this situation, a constant magnitude rotating field will be generated.

The 3-phase winding consists of 3 separate windings spaced 120° apart around the surface of the machine.

Consider a simple 3-phase stator containing three coils, each 120° apart. Such a winding will produce only one north and one south magnetic pole; therefore, this motor would be called a two-pole motor.



Assume that the currents in three coils are:

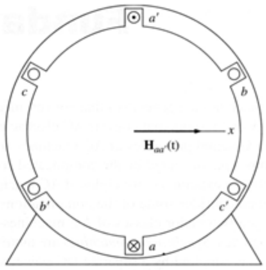
$$\begin{cases} i_{aa'}(t) = I_M \sin \omega t \\ i_{bb'}(t) = I_M \sin\left(\omega t - 120^0\right) \\ i_{cc'}(t) = I_M \sin\left(\omega t - 240^0\right) \end{cases}$$

The directions of currents are indicated.

Therefore, the current through the coil *aa'* produces the magnetic field intensity

$$H_{aa'}(t) = H_M \sin \omega t \angle 0°$$

where the magnitude of the magnetic field intensity is changing over time, while 0° is the spatial angle of the magnetic field intensity vector. The direction of the field can be determined by the right-hand rule.



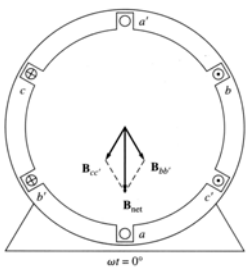
Note, that while the magnitude of the magnetic field intensity $H_{aa'}$ varies sinusoidally over time, its direction is always constant. Similarly, the magnetic fields through two other coils are

$$H_{bb'}(t) = H_M \sin\left(\omega t - 120°\right)\angle 120°$$
$$H_{cc'}(t) = H_M \sin\left(\omega t - 240°\right)\angle 240°$$

The magnetic flux densities resulting from these magnetic field intensities can be found from

$$B = \mu H$$

$$B_{aa'}(t) = \mu H_M \sin \omega t \angle 0°$$
$$B_{bb'}(t) = \mu H_M \sin\left(\omega t - 120°\right)\angle 120°$$
$$B_{cc'}(t) = \mu H_M \sin\left(\omega t - 240°\right)\angle 240°$$



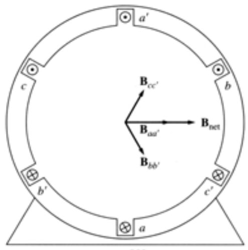
At the time $t = 0$ ($\omega t = 0$) :

$$B_{aa'}(t) = 0$$
$$B_{bb'}(t) = \mu H_M \sin\left(-120°\right)\angle 120°$$
$$B_{cc'}(t) = \mu H_M \sin\left(-240°\right)\angle 240°$$

The total magnetic field from all three coils added together will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'} = 0 + \left(-\frac{\sqrt{3}}{2}\mu H_M\right)\angle 120° + \left(\frac{\sqrt{3}}{2}\mu H_M\right)\angle 240° = 1.5\mu H_M \angle -90°$$

At the time $t = \pi/2\omega$ ($\omega t = 90°$) :



$\omega t = 90°$

$$B_{aa'}(t) = \mu H_M \angle 0°$$
$$B_{bb'}(t) = \mu H_M \sin(-30°)\angle 120° = -0.5\mu H_M \angle 120°$$
$$B_{cc'}(t) = \mu H_M \sin(-150°)\angle 240° = -0.5\mu H_M \angle 240°$$

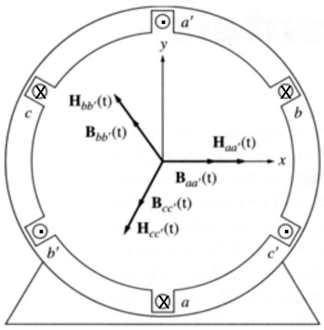
The total magnetic field from all three coils added together will be

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'} = \mu H_M \angle 0° + \left(-0.5\mu H_M\right)\angle 120° + \left(-0.5\mu H_M\right)\angle 240°$$
$$= 1.5\mu H_M \angle 0°$$

We note that the magnitude of the magnetic field is constant but its direction changes.

Therefore, **the constant magnitude magnetic field is rotating in a counterclockwise direction**.

The magnetic flux density in the stator at any arbitrary moment is given by



$$B_{net}(t) = B_{aa'}(t) + B_{bb'}(t) + B_{cc'}(t)$$
$$= \mu H_M \sin \omega t \angle 0° + \mu H_M \sin\left(\omega t - 120°\right)\angle 120°$$
$$+ \mu H_M \sin\left(\omega t - 240°\right)\angle 240°$$

Each vector can be represented as a sum of x and y components:

$$B_{net}(t) = \mu H_M \sin \omega t \ \hat{x}$$
$$-0.5\mu H_M \sin\left(\omega t - 120°\right)\hat{x} + \frac{\sqrt{3}}{2}\mu H_M \sin\left(\omega t - 120°\right)\hat{y}$$
$$-0.5\mu H_M \sin\left(\omega t - 240°\right)\hat{x} - \frac{\sqrt{3}}{2}\mu H_M \sin\left(\omega t - 240°\right)\hat{y}$$

Which can be rewritten in form

$$B_{net}(t) = \left[\mu H_M \sin\omega t - 0.5\mu H_M \sin\left(\omega t - 120°\right) - 0.5\mu H_M \sin\left(\omega t - 240°\right)\right]\hat{x}$$
$$+\left[\frac{\sqrt{3}}{2}\mu H_M \sin\left(\omega t - 120°\right) - \frac{\sqrt{3}}{2}\mu H_M \sin\left(\omega t - 240°\right)\right]\hat{y}$$
$$= \left[\mu H_M \sin\omega t + \frac{1}{4}\mu H_M \sin\omega t + \frac{\sqrt{3}}{4}\mu H_M \cos\omega t + \frac{1}{4}\mu H_M \sin\omega t - \frac{\sqrt{3}}{4}\mu H_M \cos\omega t\right]\hat{x}$$
$$+\left[-\frac{\sqrt{3}}{4}\mu H_M \sin\omega t - \frac{3}{4}\mu H_M \cos\omega t + \frac{\sqrt{3}}{4}\mu H_M \sin\omega t - \frac{3}{4}\mu H_M \cos\omega t\right]\hat{y}$$

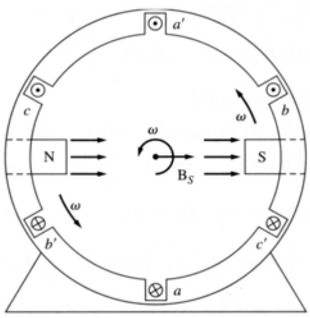
Finally

$$\boxed{B_{net}(t) = \left[1.5\mu B_M \sin\omega t\right]\hat{x} - \left[1.5\mu B_M \cos\omega t\right]\hat{y}}$$

The net magnetic field has a constant magnitude and rotates counterclockwise at the angular velocity ω.

The stator rotating magnetic field can be represented as a north pole and a south pole. These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of current.

Therefore, the mechanical speed of rotation of the magnetic field equals to the electrical frequency.



$$\left.\begin{aligned} f_e\,[Hz] &= f_m\,[rps] \\ \omega_e[rad/s] &= \omega_m\,[rad/s] \end{aligned}\right\}\ \textit{two poles}$$

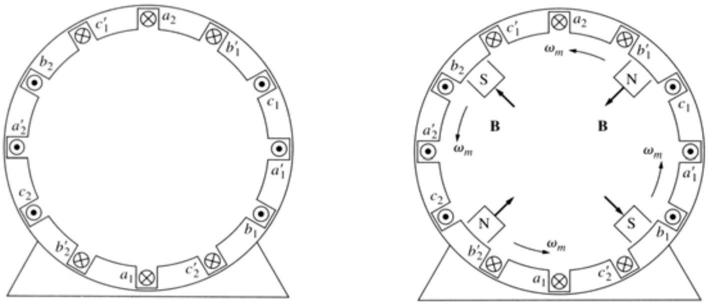
The magnetic field passes the windings of a two-pole stator in the following counterclockwise sequence: a-c’-b-a’-c-b’.

What if 3 additional windings will be added?

The new sequence will be: a-c’-b-a’-c-b’-a-c’-b-a’-c-b’ and, when 3-phase current is applied to the stator, **two north** poles and **two south** poles will be produced. In this winding, a pole moves only halfway around the stator.

The relationship between the electrical angle θ_e (current's phase change) and the mechanical angle θ_m (at which the magnetic field rotates) in this situation is:

$$\theta_e = 2\theta_m$$



Therefore, for a four-pole stator:

$$\left.\begin{array}{l} f_e\,[Hz] = 2\,f_m\,[rps] \\ \omega_e[rad/s] = 2\omega_m\,[rad/s] \end{array}\right\}\ \textit{four poles}$$

For an **AC machine with *P* poles** in its stator:

$$\boxed{\theta_e = \frac{P}{2}\theta_m}$$

$$\boxed{f_e = \frac{P}{2}f_m}$$

$$\boxed{\omega_e = \frac{P}{2}\omega_m}$$

Relating the electrical frequency to the motors speed rpm:

$$\boxed{f_e = \frac{P}{120}n_m}$$

If the current in any two of the three coils is swapped, the direction of magnetic field rotation will be reversed.

Therefore, to change the direction of rotation of an AC motor, we need to switch the connections of any two of the three coils.

In this situation, the net magnetic flux density in the stator is

$$\begin{aligned} B_{net}(t) &= B_{aa'}(t) + B_{bb'}(t) + B_{cc'}(t) \\ &= B_M \sin \omega t \angle 0° + B_M \sin\left(\omega t - 240°\right) \angle 120° + B_M \sin\left(\omega t - 120°\right) \angle 240° \end{aligned}$$

$$\begin{aligned} B_{net}(t) = B_M \sin \omega t\ \hat{x} &- \left[0.5 B_M \sin\left(\omega t - 240°\right)\right]\hat{x} + \left[\frac{\sqrt{3}}{2} B_M \sin\left(\omega t - 240°\right)\right]\hat{y} \\ &- \left[0.5 B_M \sin\left(\omega t - 120°\right)\right]\hat{x} + \left[\frac{\sqrt{3}}{2} B_M \sin\left(\omega t - 120°\right)\right]\hat{y} \end{aligned}$$

$$\begin{aligned} B_{net}(t) = &\left[B_M \sin \omega t - 0.5 B_M \sin\left(\omega t - 240°\right) - 0.5 B_M \sin\left(\omega t - 120°\right)\right]\hat{x} \\ &+ \left[\frac{\sqrt{3}}{2} B_M \sin\left(\omega t - 240°\right) + \frac{\sqrt{3}}{2} B_M \sin\left(\omega t - 120°\right)\right]\hat{y} \end{aligned}$$

Therefore:

$$\begin{aligned} B_{net}(t) = &\left[B_M \sin \omega t + \frac{1}{4} B_M \sin \omega t - \frac{\sqrt{3}}{4} B_M \cos \omega t + \frac{1}{4} B_M \sin \omega t - \frac{\sqrt{3}}{4} B_M \cos \omega t\right]\hat{x} \\ &+ \left[-\frac{\sqrt{3}}{4} B_M \sin \omega t + \frac{3}{4} B_M \cos \omega t + \frac{\sqrt{3}}{4} B_M \sin \omega t + \frac{3}{4} B_M \cos \omega t\right]\hat{y} \end{aligned}$$

$$\boxed{B_{net}(t) = \left[1.5 \mu B_M \sin \omega t\right]\hat{x} + \left[1.5 \mu B_M \cos \omega t\right]\hat{y}}$$

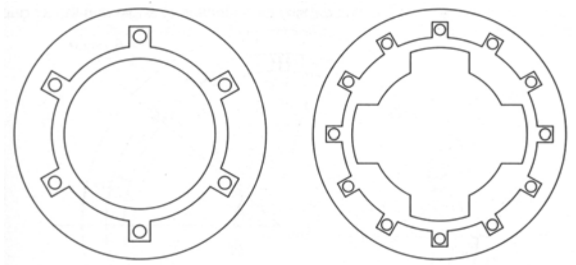
The net magnetic field has a constant magnitude and rotates clockwise at the angular velocity ω.

Switching the currents in two stator phases reverses the direction of rotation in an AC machine.

Magnetomotive force and flux distribution on an AC machine

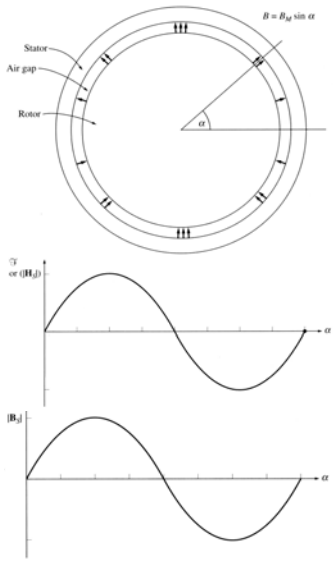
In the previous discussion, we assumed that the flux produced by a stator inside an AC machine behaves the same way it does in a vacuum. However, in real machines, there is a ferromagnetic rotor in the center with a small gap between a rotor and a stator.

A rotor can be cylindrical (such machines are said to have **non-salient poles**), or it may have pole faces projecting out from it (**salient poles**). We will restrict our discussion to non-salient pole machines only (cylindrical rotors).



The reluctance of the air gap is much higher than the reluctance of either the rotor or the stator; therefore, **the flux density vector *B* takes the shortest path across the air gap**: it will be perpendicular to both surfaces of rotor and stator.

To produce a sinusoidal voltage in this machine, the magnitude of the flux density vector *B* must vary sinusoidally along the surface of the air gap. Therefore, the magnetic field intensity (and the mmf) will vary sinusoidally along the air gap surface.

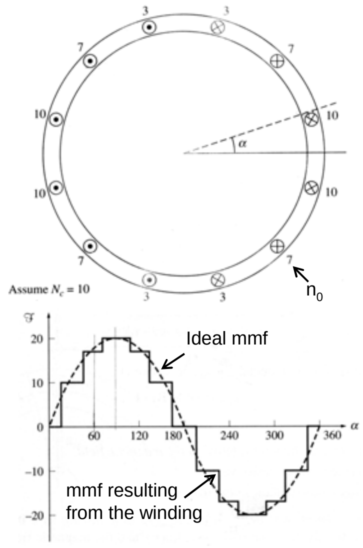


One obvious way to achieve a sinusoidal variation of mmf along the air gap surface would be to distribute the turns of the winding that produces the mmf in closely spaced slots along the air gap surface and vary the number of conductors in each slot sinusoidally, according to:

$$n_c = N_c \cos\alpha$$

where N_c is the number of conductors at the angle of 0^0 and α is the angle along the surface.

However, in practice, only a finite number of slots and integer numbers of conductors are possible. As a result, real mmf will approximate the ideal mmf if this approach is taken.



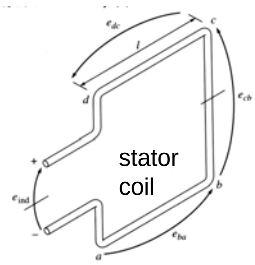
Induced voltage in AC machines

Just as a 3-phase set of currents in a stator can produce a rotating magnetic field, a rotating magnetic field can produce a 3-phase set of voltages in the coils of a stator.

The induced voltage in a single coil on a two-pole stator

Assume that a rotor with a sinusoidally distributed magnetic field rotates in the center of a stationary coil.

We further assume that the magnitude of the flux density B in the air gap between the rotor and the stator varies sinusoidally with mechanical angle, while its direction is always radially outward.

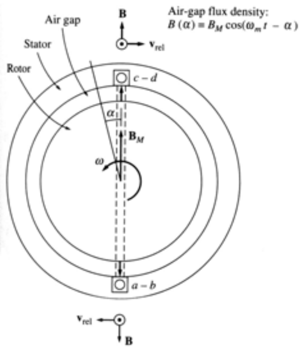


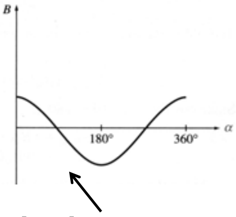
Note, that this is an ideal flux distribution.

The magnitude of the flux density vector at a point around the **rotor** is

$$B = B_M \cos\alpha \qquad (6.19.1)$$

Where α is the angle from the direction of peak flux intensity.





Flux density in a gap

Since the rotor is rotating within the stator at an angular velocity ω_m, the magnitude of the flux density vector at any angle α around the **stator** is

$$\boxed{B = B_M \cos\left(\omega t - \alpha\right)}$$

The voltage induced in a wire is

$$e_{ind} = \left(v \times B\right) \cdot l$$

Here v is the velocity of the wire **relative to the magnetic field**
B is the magnetic flux density vector
l is the length of conductor **in the magnetic field**

However, this equation was derived for a moving wire in a stationary magnetic field. In our situation, the wire is stationary and the magnetic field rotates. Therefore, the equation needs to be modified: we need to change reference such way that the field appears as stationary.

The total voltage induced in the coil is a sum of the voltages induced in each of its four sides. These voltages are:

1. Segment *ab*: $\alpha = 180^0$; assuming that *B* is radially outward from the rotor, the angle between *v* and *B* is 90^0, so

$$e_{ba} = (v \times B) \cdot I = -vB_M l \cos(\omega_m t - 180°)$$

2. Segment *bc*: the voltage will be zero since the vectors (*v* x *B*) and *l* are perpendicular.

$$e_{cb} = (v \times B) \cdot I = 0$$

3. Segment *cd*: $\alpha = 0^0$; assuming that *B* is radially outward from the rotor, the angle between *v* and *B* is 90^0, so

$$e_{dc} = (v \times B) \cdot I = vB_M l \cos(\omega_m t)$$

4. Segment *da*: the voltage will be zero since the vectors (*v* x *B*) and *l* are perpendicular.

$$e_{ad} = (v \times B) \cdot I = 0$$

Therefore, the total voltage on the coil is:

$$e_{ind} = e_{ba} + e_{dc} = -vB_M l \cos(\omega_m t - 180°) + vB_M l \cos \omega_m t$$
$$= \{\cos\theta = -\cos(\theta)\} = 2vB_M l \cos \omega_m t$$

Since the velocity of the end conductor is $v = r\omega_m$

Then: $e_{ind} = 2rlB_M \omega_m \cos \omega_m t$

The flux passing through a coil is $\phi = 2rlB_M$

Therefore: $e_{ind} = \phi\omega_m \cos \omega_m t$

Finally, if the stator coil has N_C turns of wire, the total induced voltage in the coil:

$$\boxed{e_{ind} = N_C \phi \omega_m \cos \omega_m t}$$

The induced voltage in a 3-phase set of coils

In three coils, each of N_C turns, placed around the rotor magnetic field, the induced in each coil will have the same magnitude and phases differing by 120^0:

$$e_{aa'}(t) = N_C \phi \omega_m \cos \omega_m t$$
$$e_{bb'}(t) = N_C \phi \omega_m \cos\left(\omega_m t - 120°\right)$$
$$e_{cc'}(t) = N_C \phi \omega_m \cos\left(\omega_m t - 240°\right)$$

A 3-phase set of currents can generate a uniform rotating magnetic field in a machine stator, and a uniform rotating magnetic field can generate a 3-phase set of voltages in such stator.

RMS voltage in a 3-phase stator

The peak voltage in any phase of a 3-phase stator is:

$$E_{\max} = N_C \phi \omega_m$$

For a 2-pole stator: $\omega_m = \omega_e = \omega = 2\pi f$

Thus: $E_{\max} = 2\pi N_C \phi f$

The rms voltage in any phase of a 2-pole 3-phase stator is:

$$\boxed{E_A = \frac{2\pi}{\sqrt{2}} N_C \phi f = \sqrt{2}\pi N_C \phi f}$$

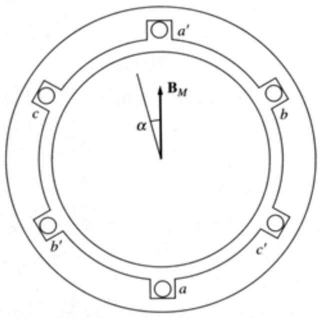
The rms voltage at the terminals will depend on the type of stator connection: if the stator is Y-connected, the terminal voltage will be $\sqrt{3}E_A$. For the delta connection, it will be just E_A.

Ex1: The peak flux density of the rotor magnetic field in a simple 2-pole 3-phase generator is 0.2 T; the mechanical speed of rotation is 3600 rpm; the stator diameter is 0.5 m; the length of its coil is 0.3 m and each coil consists of 15 turns of wire. The machine is Y-connected.

a) What are the 3-phase voltages of the generator as a function of time?
b) What is the rms phase voltage of the generator?
c) What is the rms terminal voltage of the generator?

Sol. The flux in this machine is given by

$$\phi = 2rlB = dlB = 0.5\cdot 0.3\cdot 0.2 = 0.03\,Wb$$



The rotor speed is

$$\omega = \frac{3600\cdot 2\pi}{60} = 377\ \frac{rad}{s}$$

a) The magnitude of the peak phase voltage is

$$E_{\max} = N_C\phi\omega = 15\cdot 0.03\cdot 377 = 169.7\ V$$

and the three phase voltages are:

$$e_{aa'}(t) = 169.7\sin\left(377t\right)$$
$$e_{bb'}(t) = 169.7\sin\left(377t - 120°\right)$$
$$e_{cc'}(t) = 169.7\sin\left(377t - 240°\right)$$

b) The rms voltage of the generator is

$$E_A = \frac{E_{\max}}{\sqrt{2}} = \frac{169.7}{\sqrt{2}} = 120\ V$$

c) For a Y-connected generator, its terminal voltage is

$$V_T = \sqrt{3}\cdot 120 = 208\ V$$

Induced torque in an AC machine

In an AC machine under normal operating conditions two magnetic fields are present: a field from the rotor and a field from the stator circuits. The interaction of these magnetic fields produces the torque in the machine.

Assuming a sinusoidal stator flux distribution peaking in the upward direction

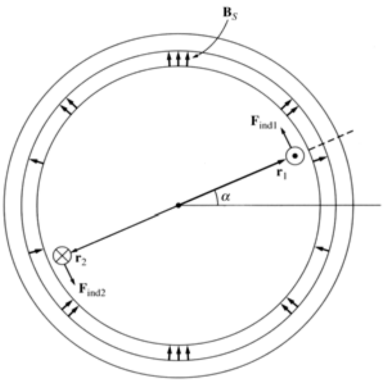
$$B_S(\alpha) = B_S \sin\alpha$$

(where B_S is the magnitude of the peak flux density) and a single coil of wire mounted on the rotor, the induced force on the first conductor (on the right) is

$$F = i(\mathbf{l} \times \mathbf{B}) = ilB_S \sin\alpha$$

The torque on this conductor is (counter-clockwise)

$$\tau_{ind,1} = \mathbf{r} \times \mathbf{F} = rilB_S \sin\alpha$$



$B_S(\alpha) = B_S \sin\alpha$

The induced force on the second conductor (on the left) is

$$F = i(\mathbf{l} \times \mathbf{B}) = ilB_S \sin\alpha$$

The torque on this conductor is (counter-clockwise)

$$\tau_{ind,2} = \mathbf{r} \times \mathbf{F} = rilB_S \sin\alpha$$

Therefore, the torque on the rotor loop is

$$\boxed{\tau_{ind} = 2rilB_S \sin\alpha}$$

We may notice the following:

1. The current i flowing in the rotor coil produces its own magnetic field H_R, whose magnitude is proportional to the current and direction can be found via the RHR.

2. The angle between the peak of the stator flux density B_S and the peak of the magnetic field intensity H_R is γ.

Furthermore,

$$\gamma = 180° - \alpha$$
$$\sin\gamma = \sin(180° - \alpha) = \sin\alpha$$

Therefore, the torque on the loop is

$$\tau_{ind} = KH_R B_S \sin\alpha$$

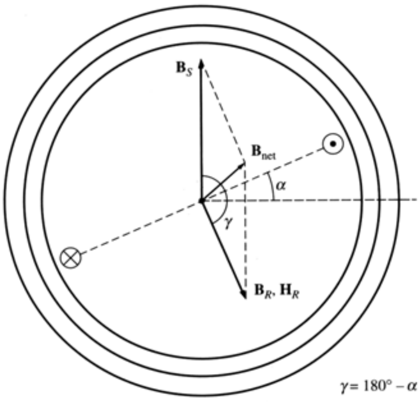
Here K is a constant dependent on the machine design. Therefore:

$$\tau_{ind} = KH_R \times B_S$$

Since $B_R = \mu H_R$

$$\boxed{\tau_{ind} = kB_R \times B_S}$$

Here $k = K/\mu$ is a constant dependent on the machine design.



The torque equation can be applied to any AC machine, not just to simple one-loop rotors. Since this equation is used for qualitative studies of torque, the constant k is not important.

Assuming no saturation, the net magnetic field is a vector sum of rotor and stator fields:

$$B_{net} = B_R + B_S$$

Combining the last equation with the torque equation, we arrive at

$$\tau_{ind} = kB_R \times (B_{net} - B_R) = k(B_R \times B_{net}) - k(B_R \times B_R)$$

Since the cross-product of any vector with itself is zero:

$$\boxed{\tau_{ind} = kB_R \times B_{net}}$$

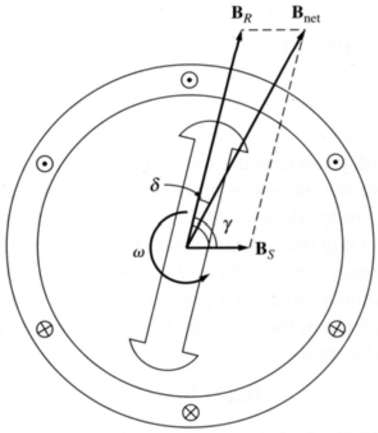
Assuming that the angle between the rotor B_R and stator B_S magnetic fields is δ:

$$\boxed{\tau_{ind} = kB_R B_{net} \sin\delta}$$

Assume that the rotor of the AC machine is rotating counter-clockwise and the configuration of magnetic fields is shown.

The combination of and the RHR shows that the torque will be clockwise, i.e. opposite to the direction of rotation of the rotor.

Therefore, this machine must be acting as a generator.



Winding insulation in AC machines

Winding insulation is of critical importance. If insulation of a motor or generator breaks down, the machine shorts out and the repair is expensive and sometimes even impossible. Most insulation failures are due to **overheating**.

To limit windings temperature, the **maximum power** that can be supplied by the machine must be limited in addition to the proper ventilation.

ROT: the life expectancy of a motor with a given type of insulation is halved for each 10^0C rise above the rated winding temperature.

AC machine power flows and losses

The **efficiency** of an AC machine is defined as

$$\boxed{\eta = \frac{P_{out}}{P_{in}} \cdot 100\%}$$

Since the difference between the input and output powers of a machine is due to the losses occurring inside it, the efficiency is

$$\boxed{\eta = \frac{P_{in} - P_{loss}}{P_{in}} \cdot 100\%}$$

Losses occurring in an AC machine can be divided into four categories:

1. Electrical or Copper losses

These losses are resistive heating losses that occur in the stator (**armature**) winding and in the rotor (**field**) winding of the machine. For a 3-phase machine, the stator copper losses and synchronous rotor copper losses are:

$$\boxed{\begin{aligned} P_{SCL} &= 3I_A^2 R_A \\ P_{RCL} &= 3I_F^2 R_F \end{aligned}}$$

Where I_A and I_F are currents flowing in each armature phase and in the field winding respectively. R_A and R_F are resistances of each armature phase and of the field winding respectively. These resistances are usually measured at normal operating temperature.

2. Core losses

These losses are the hysteresis losses and eddy current losses. They vary as B^2 (flux density) and as $n^{1.5}$ (speed of rotation of the magnetic field).

3. Mechanical losses

There are two types of mechanical losses: friction (friction of the bearings) and windage (friction between the moving parts of the machine and the air inside the casing). These losses are often lumped together and called the no-load rotational loss of the machine. They vary as the cube of rotation speed n^3.

4. Stray (miscellaneous) losses

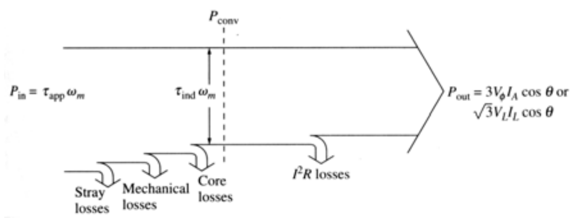
These are the losses that cannot be classified in any of the previous categories. They are usually due to inaccuracies in modeling. For many machines, stray losses are assumed as 1% of full load.

The power-flow diagram

On of the most convenient technique to account for power losses in a machine is the **power-flow diagram**.

<u>AC generator:</u>

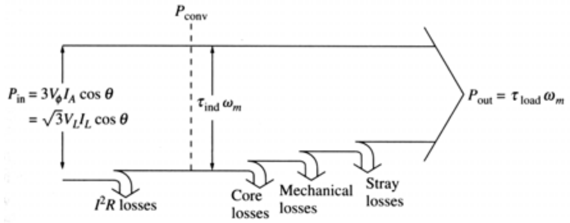
The mechanical power is input, and then all losses but cupper are subtracted. The remaining power P_{conv} is ideally converted to electricity:



$$P_{conv} = \tau_{ind}\omega_m$$

<u>AC motor:</u>

Power-flow diagram is simply reversed.



Voltage regulation

Voltage regulation (VR) is a commonly used figure of merit for generators:

$$\boxed{VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \cdot 100\%}$$

Here V_{nl} and V_{fl} are the no-load full-load terminal voltages of the generator. VR is a rough measure of the generator's voltage-current characteristic. A small VR (desirable) implies that the generator's output voltage is more constant for various loads.

Speed regulation

Speed regulation (SR) is a commonly used figure of merit for motors:

$$\boxed{SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \cdot 100\%}$$

$$\boxed{SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \cdot 100\%}$$

Here n_{nl} and n_{fl} are the no-load full-load speeds of the motor. SR is a rough measure of the motor's torque-speed characteristic. A positive SR implies that a motor's speed drops with increasing load. The magnitude of SR reflects a steepness of the motor's speed-torque curve.